EPSILON INDIA 2024

Exploration Problems for Epsilon India 2024

- Please answer the questions below in a separate sheet. Make sure you write the question number and answers clearly.
- Please take your time to write down the steps as we are interested in your approach rather than just your answer. There is no time limit. This will help us to determine if you have the critical reasoning skills and mathematical foundation that will make you a good fit for the camp.
- Do make an attempt to try out all the questions! It is not a problem, if you are unable to solve all the questions correctly. Our Faculty will review your answers and will be able to determine your strength/and provide suggestions on topics to study prior to the camp, if you are selected.
- Remember, that answers to these questions may not be quick and easy like problems you see in your homework or even some mathematics competitions. You will have to spend time thinking about them deeply. Most of our students work on these problems over a few days and revisit the solutions to improve on them. This will greatly improve the chances of doing well on the test and getting selected. These questions are meant to exercise your cognitive skills, which will equip you to solve problems later in your career that require perseverance and critical thinking.
- The work has to be completed on your own. If you seek help, you will be at a huge disadvantage as you will be barred from attending Epsilon India permanently. In the off chance that a student gets through the admission process and attends the camp, we have noted such students really struggle through the camp and it affects their confidence negatively. Fees will not be refunded.
- We hope you struggle but enjoy the process of doing these problems!

Sending the forms to us:

Priority given to candidates applying before February 2nd, 2024. Registrations open till spots left. Once you have completed the 'Algebra Assessment' and 'Exploration Problems':

- Please go to the 'How to Apply' section of www.epsilonindia.org and follow the instructions to complete the Application process on our student app. Parents can help with the Application process but the applicant has to work independently on the 'Algebra Assessment' and 'Exploration Problems'. Note that the completed tests must be scanned and uploaded as a single pdf into the student app.
- Please also send the scanned work on both the 'Algebra Assessment' and 'Exploration Problems' as attachments to an email to epsiloncampindia@gmail.com
- It is very important that you do not send us individual photos of the problems in the test. The Algebra Assessment must be scanned as one pdf and the Exploration problems has to be scanned as another pdf document and then emailed to us and also uploaded into the student app.

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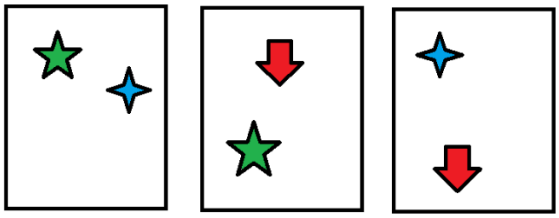
1) There is a deck of *distinct* cards. Every card in the deck has an equal number of objects in it. The objects are placed in such a way that in any given two cards, there is exactly one object that is common in them. For instance, in the example shown below, there are exactly two objects in every card, three cards in total and three objects in total. If you take the first two cards, you have only the star common. In the second and third card, only the arrow is common and in the first and third card, only the four-pointed star is common.
Can you explore this question for other numbers? For instance, can we have a deck of 5 cards or 6 cards or 7 cards, or any given number of cards? Can we have a deck with any number of objects on each card? Is there any relationship between the number of objects in every card and the number of cards? Explore and come up with as many observations as you can.



Total no. of objects	Total no. of cards	No. of objects in each card
3	3	2

2) There is a machine that takes any number as input and gives an output after going through a particular *function*. What the *function* does is - it counts the number of 0's first, then the 1's, then the 2's, and so on up to the 9's and returns the count first followed by the digit that it is counting. E.g. If the input is 1032, the output will be 10111213. This means, there will be 1 zero, 1 one, 1 two and 1 three. We will represent this as $f(1032) = 10111213$ where 1032 is the input, $f(\ )$ will represent the function and 10111213 represents the output. This is the first iteration (step). In the second iteration, 10111213 will be the input and the output will be 10511213. In the third iteration, 10511213 will be the input and we will get some other output. This process will continue.

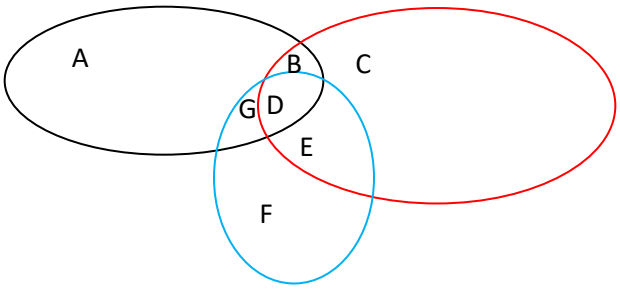
If you continue this process for $f(1)$ for a few steps (say, 10 or 15 steps), you will see that you will land up in a number and get stuck at it. We will call this number a *constant*.

The questions that you need to explore are:

i. What will happen if you do the process for $f(2)$? Will it also reach a constant? Why do you think it will reach *that* particular constant? Give at least 5 other numbers which when given as the input will reach the same constant that you get for $f(2)$.

ii. At most, how many such constants do you think can exist? We don't expect you to calculate for every number and get the constants for every input, that would be impossible. What we expect is, can you give a logical argument as to *how many* constants *can* exist? E.g. You may feel that there can be 10 to 15 different constants for various inputs. In that case, you need to give a *reason* as to why you feel there cannot be more than 15 constants.

3) The black circle is the set of numbers from 400 to 800, that are divisible by 5, the red one represents the set of numbers that are divisible by 7 and blue represents the ones that are NOT divisible by 11. Based on this information, answer the following questions:



a. Which alphabet will have maximum numbers and why?

b. Give three numbers that will come under:
   a. B, G and E.
   b. Will there be numbers that will not come under any of these sets? In other words, will there be numbers from 400 to 800 that are not represented by any of these alphabets?

4) Consider the number 15. It can be written as sum of consecutive numbers as 7+8 or 1+2+3+4+5. Can we do this for other positive integers as well?

Is it possible to express any positive integer as the sum of consecutive positive integers? Are there other integers like 15 that can be expressed as the sum of consecutive positive integers in more than 1 way? Is it possible to figure out a formula for finding the *number of ways* to express a number as the sum of consecutive positive integers? Share your answers and working in detail.

5) If we list down all the 5-digit numbers,
   i. How many of them will contain the digit 3?
   ii. How many times the digit 3 will appear in all the 5-digit numbers?

6) Seven thieves steal a certain number of diamonds. On the way back home, they all decide to take a nap under a tree. While the others are asleep, two of them wake up and decide to divide the loot amongst just the two of them equally. At the end of the division process, they find that there is one diamond left. By that time, one more thief wakes up. On his demand for his share, all three of them try to divide the loot equally amongst them. They find two diamonds extra after each one gets equal share. The fourth thief wakes up and they do the division once again. Now there are three diamonds left. With the fifth thief also waking up, there are four diamonds left after the division. The sixth thief also wakes up and they do the same process only to find that there are five diamonds left. Finally, when the seventh thief wakes up, they were able to divide the loot amongst themselves equally. Can you find the number of diamonds? How many solutions exist for this problem, and why?

7) There are 10 dice stacked one on top of the other. For the topmost dice, there are five faces visible, and for every dice below it there are 4 faces visible. If the number on the upper face of the topmost dice is 5, find out the sum of all the numbers on the faces that are not visible from any side after they are stacked up one over the other? The order of numbers on the dice (in the image below) would be helpful to solve the problem.